**Abstract**

This vignette provides an overview of calculating portfolio returns through time with an emphasis on the math used to develop the `Return.portfolio` function in **PerformanceAnalytics**. We first introduce some basic definitions, then give simple examples of computing portfolio returns in a prices and shares framework as well as a returns and weights framework. We then introduce `Return.portfolio` and demonstrate the function with a few examples.



# 1 Basic definitions

Suppose we have a portfolio of $N$ assets. The value of asset $i$, $V_i$, in the portfolio is defined as

$$V_i = \lambda_i * P_i$$

where:

$$\lambda_i \text{ is the number of shares of asset } i$$
$$P_i \text{ is the price of asset } i$$

The total portfolio value, $V_p$, is defined as

$$V_P = \sum_{i=1}^{N} V_i$$

The weight of asset $i$, $w_i$, in the portfolio is defined as

$$w_i = V_i / V_P$$

where:

$$V_i \text{ is the value of asset } i$$
$$V_P \text{ is the total value of the portfolio}$$

The portfolio return at time $t$, $R_t$, is defined as

$$R_t = \frac{V_{p_t} - V_{p_{t-1}}}{V_{p_{t-1}}}$$

$$V_{p_t} \text{ is the portfolio value at time } t$$

## 2 Simple Example: Prices and Shares Framework

Suppose we have a portfolio of $N = 2$ assets, asset A and asset B. The prices for assets A and B are given as

```
> prices = cbind(c(5, 7, 6, 7),
+                c(10, 11, 12, 8))
> dimnames(prices) = list(paste0("t",0:3), c("A", "B"))
> prices
```

```
   A  B
t0 5 10
t1 7 11
t2 6 12
t3 7  8
```

We wish to form an equal weight portfolio, that is, form a portfolio where

$$w_i = \frac{1}{N} \text{ for } i \in 1, \ldots, N.$$

Let $V_{P0} = 1000$ be the portfolio value at $t_0$.
Step 1: Compute the number of shares of each asset to purchase.

$$\begin{aligned} w_i &= \frac{V_i}{V_P} \\ &= \frac{\lambda_i * P_i}{V_P} \end{aligned}$$

Solve for $\lambda_i$.

$$\lambda_i = \frac{w_i * V_P}{P_i}$$

$$\begin{aligned} \lambda_A &= \frac{w_A * V_P 0}{P_A} = \frac{0.5 * \$1000}{\$5} = 100 \\ \lambda_B &= \frac{w_B * V_P 0}{P_B} = \frac{0.5 * \$1000}{\$10} = 50 \end{aligned}$$

```
> V_P0 = 1000
> N = ncol(prices)
> w = rep(1 / N, N)
> lambda = w * V_P0 / prices["t0",]
> lambda
```

```
  A   B
100  50
```

Step 2: Compute the asset value and portfolio value for $t \in 0, \ldots, 3$.

```
> # Compute the value of the assets
> V_assets <- matrix(0, nrow(prices), ncol(prices), dimnames=dimnames(prices))
> for(i in 1:nrow(prices)){
+   V_assets[i,] = prices[i,] * lambda
+ }
> V_assets

     A   B
t0 500 500
t1 700 550
t2 600 600
t3 700 400

> # Compute the value of the portfolio
> V_P = rowSums(V_assets)
> V_P

  t0   t1   t2   t3
1000 1250 1200 1100
```

Step 3: Compute the portfolio returns for $t \in 1, \ldots, 3$.

```
> # Compute the portfolio returns
> R_t = diff(V_P) / V_P[1:3]
> R_t

         t1          t2          t3
 0.25000000 -0.04000000 -0.08333333
```

Step 4: Compute the weights of each asset in the portfolio for $t \in 0, \ldots, 3$

```
> weights = V_assets / V_P
> weights

           A         B
t0 0.5000000 0.5000000
t1 0.5600000 0.4400000
t2 0.5000000 0.5000000
t3 0.6363636 0.3636364
```

We have shown that calculating portfolio weights, values, and returns is simple in a prices and shares framework. However, calculating these metrics becomes more challenging in a weights and returns framework.

# 3 Example: Weights and Returns Framework

We will use the monthly returns during 1997 of the first 5 assets in the edhec dataset for the following example.

```
> library(PerformanceAnalytics)
> data(edhec)
> R = edhec["1997", 1:5]
> colnames(R) = c("CA", "CTAG", "DS", "EM", "EMN")
> R

               CA    CTAG      DS      EM    EMN
1997-01-31 0.0119  0.0393  0.0178  0.0791 0.0189
1997-02-28 0.0123  0.0298  0.0122  0.0525 0.0101
1997-03-31 0.0078 -0.0021 -0.0012 -0.0120 0.0016
1997-04-30 0.0086 -0.0170  0.0030  0.0119 0.0119
1997-05-31 0.0156 -0.0015  0.0233  0.0315 0.0189
1997-06-30 0.0212  0.0085  0.0217  0.0581 0.0165
1997-07-31 0.0193  0.0591  0.0234  0.0560 0.0247
1997-08-31 0.0134 -0.0473  0.0147 -0.0066 0.0017
1997-09-30 0.0122  0.0198  0.0350  0.0229 0.0202
1997-10-31 0.0100 -0.0098 -0.0064 -0.0572 0.0095
1997-11-30 0.0000  0.0133  0.0054 -0.0378 0.0041
1997-12-31 0.0068  0.0286  0.0073  0.0160 0.0066
```

Suppose that on 1996-12-31 we wish to form an equal weight portfolio such that the weight for asset $i$ is given as:

$$w_i = \frac{1}{N} \quad \text{for } i \in 1, \ldots, N$$

where $N$ is equal to the number of assets.

```
> N = ncol(R)
> weights = xts(matrix(rep(1 / N, N), 1), as.Date("1996-12-31"))
> colnames(weights) = colnames(R)
> weights

            CA CTAG  DS  EM EMN
1996-12-31 0.2  0.2 0.2 0.2 0.2
```

There are two cases we need to consider when calculating the beginning of period (bop) value.

Case 1: The beginning of period $t$ is a rebalancing event. For example, the rebalance weights at the end of `1996-12-31` take effect at the beginning of `1997-01-31`. This means that the beginning of `1997-01-31` is considered a rebalance event.

The beginning of period value for asset $i$ at time $t$ is given as

$$V_{bop_{t,i}} = w_i * V_{t-1}$$

where $w_i$ is the weight of asset $i$ and $V_{t-1}$ is the end of period (eop) portfolio value of the prior period.

Case 2: The beginning of period $t$ is not a rebalancing event.

$$V_{bop_{t,i}} = V_{eop_{t-1,i}}$$

where $V_{eop_{t-1,i}}$ is the end of period value for asset $i$ from the prior period.

The end of period value for asset $i$ at time $t$ is given as

$$V_{eop_{t,i}} = (1 + R_{t,i}) * V_{bop_{t,i}}$$

Here we demonstrate this and compute values for the periods 1 and 2. For the first period, $t = 1$, we need an initial value for the portfolio value. Let $V_0 = 1$ denote the initial portfolio value. Note that the initial portfolio value can be any arbitrary number. Here we use $V_0 = 1$ for simplicity.

```
> V_0 = 1
> bop_value = eop_value = matrix(0, 2, ncol(R))
```

Compute the values for $t = 1$.

```
> t = 1
> bop_value[t,] = coredata(weights) * V_0
> eop_value[t,] = coredata(1 + R[t,]) * bop_value[t,]
```

Now compute the values for $t = 2$.

```
> t = 2
> bop_value[t,] = eop_value[t-1,]
> eop_value[t,] = coredata(1 + R[t,]) * bop_value[t,]
```

It is seen that the values for the rest of the time periods can be computed by iterating over $t \in 1, \ldots, T$ where $T = 12$ in this example.

The weight of asset $i$ at time $t$ is calculated as

$$w_{t,i} = \frac{V_{t,i}}{\sum_{i=0}^{N} V_{t,i}}$$

Here we compute both the beginning and end of period weights.

```
> bop_weights = eop_weights = matrix(0, 2, ncol(R))
> for(t in 1:2){
+   bop_weights[t,] = bop_value[t,] / sum(bop_value[t,])
+   eop_weights[t,] = eop_value[t,] / sum(eop_value[t,])
+ }
> bop_weights

         [,1]      [,2]      [,3]      [,4]      [,5]
[1,] 0.200000 0.2000000 0.2000000 0.2000000 0.2000000
[2,] 0.195839 0.2011419 0.1969808 0.2088446 0.1971937

> eop_weights

          [,1]      [,2]      [,3]      [,4]      [,5]
[1,] 0.1958390 0.2011419 0.1969808 0.2088446 0.1971937
[2,] 0.1936464 0.2023282 0.1947562 0.2147071 0.1945622
```

The portfolio returns at time $t$ are calculated as

$$R_{P_t} = \frac{V_t - V_{t-1}}{V_{t-1}}$$

```
> V = c(V_0, rowSums(eop_value))
> R_P = diff(V) / V[1:2]
> R_P

[1] 0.03340000 0.02376201
```

The contribution of asset $i$ at time $t$ is calculated as

$$contribution_{t,i} = \frac{V_{eop_{t,i}} - V_{bop_{t,i}}}{\sum_{i=1}^{N} V_{bop_{t,i}}}$$

```
> contribution = matrix(0, 2, ncol(R))
> for(t in 1:2){
+   contribution[t,] = (eop_value[t,] - bop_value[t,]) / sum(bop_value[t,])
+ }
> contribution

            [,1]        [,2]        [,3]       [,4]        [,5]
[1,] 0.002380000 0.007860000 0.003560000 0.01582000 0.003780000
[2,] 0.002408819 0.005994027 0.002403166 0.01096434 0.001991657
```

Note that contribution can also be calculated as

$$contribution_{t,i} = R_{t,i} * w_{t,i}$$

# 4 Return.portfolio Examples

```
> args(Return.portfolio)

function (R, weights = NULL, wealth.index = FALSE, contribution = FALSE,
    geometric = TRUE, rebalance_on = c(NA, "years", "quarters",
        "months", "weeks", "days"), value = 1, verbose = FALSE,
    ...)
NULL
```

If no `weights` are specified, then an equal weight portfolio is computed. If `rebalance_on=NA` then a buy and hold portfolio is assumed. See `?Return.portfolio` for a detailed explanation of the function and arguments.

```
> # Equally weighted, buy and hold portfolio returns
> Return.portfolio(R)

           portfolio.returns
1997-01-31       0.033400000
1997-02-28       0.023762011
1997-03-31      -0.001413340
1997-04-30       0.003678107
1997-05-31       0.017767315
1997-06-30       0.025914372
1997-07-31       0.036969968
```

```
1997-08-31      -0.005005540
1997-09-30       0.022080944
1997-10-31      -0.012352423
1997-11-30      -0.003843547
1997-12-31       0.012936194
```

```
> # Equally weighted, rebalanced quarterly portfolio returns
> Return.portfolio(R, rebalance_on="quarters")
```

```
           portfolio.returns
1997-01-31       0.033400000
1997-02-28       0.023762011
1997-03-31      -0.001413340
1997-04-30       0.003680000
1997-05-31       0.017660872
1997-06-30       0.025452430
1997-07-31       0.036500000
1997-08-31      -0.005136602
1997-09-30       0.022049167
1997-10-31      -0.010780000
1997-11-30      -0.002621013
1997-12-31       0.012985944
```

```
> # Equally weighted, rebalanced quarterly portfolio returns.
> # Use verbose=TRUE to return additional information
> # including asset values and weights
> Return.portfolio(R, rebalance_on="quarters", verbose=TRUE)
```

```
$returns
           portfolio.returns
1997-01-31       0.033400000
1997-02-28       0.023762011
1997-03-31      -0.001413340
1997-04-30       0.003680000
1997-05-31       0.017660872
1997-06-30       0.025452430
1997-07-31       0.036500000
1997-08-31      -0.005136602
1997-09-30       0.022049167
1997-10-31      -0.010780000
1997-11-30      -0.002621013
1997-12-31       0.012985944

$contribution
                    CA          CTAG            DS           EM          EMN
1997-01-31 0.002380000  0.0078600000  0.0035600000  0.015820000 0.0037800000
1997-02-28 0.002408819  0.0059940275  0.0024031662  0.010964341 0.0019916567
1997-03-31 0.001510442 -0.0004248891 -0.0002337074 -0.002576485 0.0003112995
1997-04-30 0.001720000 -0.0034000000  0.0006000000  0.002380000 0.0023800000
1997-05-31 0.003135294 -0.0002938187  0.0046568428  0.006351596 0.0038109577
```

| | | | | | |
|---|---|---|---|---|---|
| 1997-06-30 | 0.004252156 | 0.0016336242 | 0.0043610924 | 0.011874480 | 0.0033310776 |
| 1997-07-31 | 0.003860000 | 0.0118200000 | 0.0046800000 | 0.011200000 | 0.0049400000 |
| 1997-08-31 | 0.002635527 | -0.0096662672 | 0.0029028423 | -0.001344834 | 0.0003361293 |
| 1997-09-30 | 0.002444218 | 0.0038748559 | 0.0070493383 | 0.004659301 | 0.0040214532 |
| 1997-10-31 | 0.002000000 | -0.0019600000 | -0.0012800000 | -0.011440000 | 0.0019000000 |
| 1997-11-30 | 0.000000000 | 0.0026626352 | 0.0010847819 | -0.007205240 | 0.0008368108 |
| 1997-12-31 | 0.001392218 | 0.0058170647 | 0.0014782579 | 0.002942265 | 0.0013561387 |

$BOP.Weight

| | CA | CTAG | DS | EM | EMN |
|---|---|---|---|---|---|
| 1997-01-31 | 0.2000000 | 0.2000000 | 0.2000000 | 0.2000000 | 0.2000000 |
| 1997-02-28 | 0.1958390 | 0.2011419 | 0.1969808 | 0.2088446 | 0.1971937 |
| 1997-03-31 | 0.1936464 | 0.2023282 | 0.1947562 | 0.2147071 | 0.1945622 |
| 1997-04-30 | 0.2000000 | 0.2000000 | 0.2000000 | 0.2000000 | 0.2000000 |
| 1997-05-31 | 0.2009804 | 0.1958792 | 0.1998645 | 0.2016380 | 0.2016380 |
| 1997-06-30 | 0.2005734 | 0.1921911 | 0.2009720 | 0.2043800 | 0.2018835 |
| 1997-07-31 | 0.2000000 | 0.2000000 | 0.2000000 | 0.2000000 | 0.2000000 |
| 1997-08-31 | 0.1966811 | 0.2043608 | 0.1974723 | 0.2037627 | 0.1977231 |
| 1997-09-30 | 0.2003458 | 0.1956998 | 0.2014097 | 0.2034629 | 0.1990818 |
| 1997-10-31 | 0.2000000 | 0.2000000 | 0.2000000 | 0.2000000 | 0.2000000 |
| 1997-11-30 | 0.2042013 | 0.2001981 | 0.2008855 | 0.1906148 | 0.2041002 |
| 1997-12-31 | 0.2047379 | 0.2033939 | 0.2025011 | 0.1838916 | 0.2054756 |

$EOP.Weight

| | CA | CTAG | DS | EM | EMN |
|---|---|---|---|---|---|
| 1997-01-31 | 0.1958390 | 0.2011419 | 0.1969808 | 0.2088446 | 0.1971937 |
| 1997-02-28 | 0.1936464 | 0.2023282 | 0.1947562 | 0.2147071 | 0.1945622 |
| 1997-03-31 | 0.1954330 | 0.2021890 | 0.1947978 | 0.2124308 | 0.1951493 |
| 1997-04-30 | 0.2009804 | 0.1958792 | 0.1998645 | 0.2016380 | 0.2016380 |
| 1997-05-31 | 0.2005734 | 0.1921911 | 0.2009720 | 0.2043800 | 0.2018835 |
| 1997-06-30 | 0.1997416 | 0.1890138 | 0.2002366 | 0.2108869 | 0.2001210 |
| 1997-07-31 | 0.1966811 | 0.2043608 | 0.1974723 | 0.2037627 | 0.1977231 |
| 1997-08-31 | 0.2003458 | 0.1956998 | 0.2014097 | 0.2034629 | 0.1990818 |
| 1997-09-30 | 0.1984151 | 0.1952691 | 0.2039618 | 0.2036323 | 0.1987216 |
| 1997-10-31 | 0.2042013 | 0.2001981 | 0.2008855 | 0.1906148 | 0.2041002 |
| 1997-11-30 | 0.2047379 | 0.2033939 | 0.2025011 | 0.1838916 | 0.2054756 |
| 1997-12-31 | 0.2034876 | 0.2065290 | 0.2013644 | 0.1844387 | 0.2041802 |

$BOP.Value

| | CA | CTAG | DS | EM | EMN |
|---|---|---|---|---|---|
| 1997-01-31 | 0.2000000 | 0.2000000 | 0.2000000 | 0.2000000 | 0.2000000 |
| 1997-02-28 | 0.2023800 | 0.2078600 | 0.2035600 | 0.2158200 | 0.2037800 |
| 1997-03-31 | 0.2048693 | 0.2140542 | 0.2060434 | 0.2271506 | 0.2058382 |
| 1997-04-30 | 0.2112921 | 0.2112921 | 0.2112921 | 0.2112921 | 0.2112921 |
| 1997-05-31 | 0.2131092 | 0.2077001 | 0.2119260 | 0.2138065 | 0.2138065 |
| 1997-06-30 | 0.2164337 | 0.2073886 | 0.2168638 | 0.2205414 | 0.2178474 |
| 1997-07-31 | 0.2213080 | 0.2213080 | 0.2213080 | 0.2213080 | 0.2213080 |
| 1997-08-31 | 0.2255792 | 0.2343873 | 0.2264866 | 0.2337012 | 0.2267743 |

```
1997-09-30 0.2286020 0.2233008 0.2298159 0.2321588 0.2271598
1997-10-31 0.2332393 0.2332393 0.2332393 0.2332393 0.2332393
1997-11-30 0.2355716 0.2309535 0.2317465 0.2198980 0.2354550
1997-12-31 0.2355716 0.2340252 0.2329980 0.2115858 0.2364204

$EOP.Value
                  CA      CTAG        DS        EM       EMN
1997-01-31 0.2023800 0.2078600 0.2035600 0.2158200 0.2037800
1997-02-28 0.2048693 0.2140542 0.2060434 0.2271506 0.2058382
1997-03-31 0.2064673 0.2136047 0.2057962 0.2244247 0.2061675
1997-04-30 0.2131092 0.2077001 0.2119260 0.2138065 0.2138065
1997-05-31 0.2164337 0.2073886 0.2168638 0.2205414 0.2178474
1997-06-30 0.2210221 0.2091514 0.2215698 0.2333548 0.2214419
1997-07-31 0.2255792 0.2343873 0.2264866 0.2337012 0.2267743
1997-08-31 0.2286020 0.2233008 0.2298159 0.2321588 0.2271598
1997-09-30 0.2313909 0.2277221 0.2378595 0.2374752 0.2317484
1997-10-31 0.2355716 0.2309535 0.2317465 0.2198980 0.2354550
1997-11-30 0.2355716 0.2340252 0.2329980 0.2115858 0.2364204
1997-12-31 0.2371735 0.2407183 0.2346988 0.2149712 0.2379808
```